DEMONSTRATING THE USEFULNESS OF THE PARTICIPATORY-ANTICIPATORY DISTINCTION

Nicora Placa
New York University
np874@nyu.edu

Martin A. Simon
New York University
msimon@nyu.edu

This paper demonstrates the explanatory power of the participatory-anticipatory distinction postulated by Tzur and Simon (2004). Using data from our current project, we show how this lens allows us to make sense of seeming inconsistencies in a student's development of a mathematical conception. More broadly, the distinction allows us to observe, analyze and conceptualize stages of learning a particular concept and, in turn, generate more thorough accounts of student learning. Furthermore, we draw attention to its usefulness in informing the design of subsequent tasks.

Keywords: Rational Numbers; Learning Trajectories; Design Experiments

Tzur and Simon (2004) postulated different levels of abstraction in the learning of a concept (see also Tzur, 2007; Tzur & Lambert, 2011). Specifically, they characterized two levels representing different learned anticipations (reviewed below). In our current project, this distinction provided a critical lens for our analyses. In this paper, we demonstrate the explanatory power of these fine-grained distinctions in accounting for student learning and, its usefulness in informing the generation of tasks.

The Problem

As we worked with Kylie, a 4th-grade student, we were struck by the following sequence. We presented her with a series of tasks designed to develop an understanding of the relationship between mixed numbers and improper fractions. In order to identify the equivalent improper fraction for a mixed number, she was shown a bar[1] made up of whole and fractional units (representing a mixed number). She partitioned the whole units into fractional parts. Having subdivided the bar, she was able to verbalize or write the equivalent improper fraction. As she worked through the sequence, she began to anticipate what the equivalent improper fraction would be *without* partitioning the bars. For example, when she was shown a quantity that was 6 and 1/4 units long and asked to express it as an improper fraction, she quickly answered, "Twenty-five fourths." Furthermore, she was able to explain her answer, "If that was all broken up into fours ..., four times six is twenty-four. Plus one is ... twenty-five fourths." She was able to respond similarly to several other tasks of this type.

We then presented the task without setting up the figure on the screen. We asked her for an improper fraction equivalent to 2 5/7. She was unable to provide a correct response. She answered, "Um …(pause) twenty-seven fifths? No — twenty-five sevenths?" How could we explain the discrepancy between what Kylie had done on the prior tasks and her inability to complete this task? She had previously shown that she could create a mixed number from the numeral representation. She could start with a mixed number represented on the screen and anticipate the equivalent improper fraction. Why was she not able to do this most recent task?

Background and Methodology

This report is based on research conducted during the second year of the five-year Measuring Approach to Rational Number (MARN) project.[2] The project is focused on two goals: (1) increasing understanding of how students learn through their activity, and (2) understanding how students can effectively learn fraction and ratio concepts based on activities grounded in measurement. The design builds upon aspects of the Elkonin–Davydov elementary curriculum (Davydov, Gorbov, Mikulina, & Savel'eva, 1995; Davydov et al., 1999) as well as research on rational number learning. In our second year, we are conducting one-on-one teaching experiments based on task sequences we have developed.

The data in this paper comes from a one-on-one teaching experiment with Kylie, a fourth grader in New York City. We began working with her in the fall of 2011 twice a week for hour-long sessions. Our team conducts an on-going analysis of the data after each session. We make inferences about her understanding and modify our trajectory for the upcoming session.

Conceptual Framework

We now review the conceptual framework we use in this research project, including the participatory/anticipatory distinction.

Abstraction from One's Activity

An emerging body of work (Tzur & Simon, 2004; Simon, Tzur, Heinz, & Kinzel, 2004; Simon et al, 2010) builds upon Piaget (2001) and Von Glaserfield's (1995) constructs of *goal-directed activity, reflection,* and *abstraction* in order to understand the process(es) of conceptual learning. *Goal-directed activity* is used to describe both the mental and physical activity of a learner. Conceiving of the activity as goal-directed supports the researcher in understanding the learner's choice of activity and attending to what the learner is focused on; the learner's goal determines her focus. *Reflection* refers to the ability of a learner to notice (consciously or not) commonalities in his or her experience. A*bstraction* is the process by which conceptual learning occurs. Abstraction involves reflection on one's goal directed activity. This reflection or noticing of commonalities in one's activity results in an anticipation, the ability to know the effect of that activity without actually engaging in it. We elaborate further on the idea of learned anticipation in the next section.

Participatory and Anticipatory Stages

Building upon the learning through activity framework,[3] Tzur and Simon (2004) postulated two stages of abstraction as a student is developing a new mathematical conception. The first stage, termed *participatory,* refers to the idea that a learner can anticipate the result of an activity. However this anticipation is limited.

> At the participatory (first) stage, the learner has learned to anticipate the effects of an activity and may also be able to explain why the effects derive from the activity. However, this knowledge is only available to the learners *in the context of the activity* through which it was developed. "In the context of the activity" means either that the learner is engaged in the activity or is somehow (e.g., chance, social interaction) prompted to use or think about the activity. (Tzur & Simon, 2004, pp. 12–13)

The second stage, which is labeled *anticipatory,* occurs when the student can anticipate the need to call upon the activity in response to a particular type of task. At this point they are not calling on the activity alone, but the activity *and* the learned anticipation of the effect of that activity.

Analysis

The sessions analyzed here occurred after our initial work with Kylie on fractions. Kylie completed a variety of tasks designed to foster the development of an understanding of a fraction as a partial measurement unit. Many of these tasks were done using the computer program JavaBars (Biddlecomb & Olive, 2000). After this preliminary work, she was able to create and identify fractional quantities, including mixed numbers. In the sessions described, Kylie worked on a series of tasks designed to foster an understanding of how to express equivalent mixed numbers and improper fractions.

Converting Improper Fractions to Mixed Numbers

The tasks created for this session used the context of measuring a beam and communicating to the owner of a hardware store the length of the beam. Using JavaBars, the student was presented with a bar (the beam) that was unmarked, a bar that represented the unit the store owner had, and a long "measuring

strip" that had been marked with fractional parts (see Figure 1). The student was asked to measure the beam with the measuring strip.

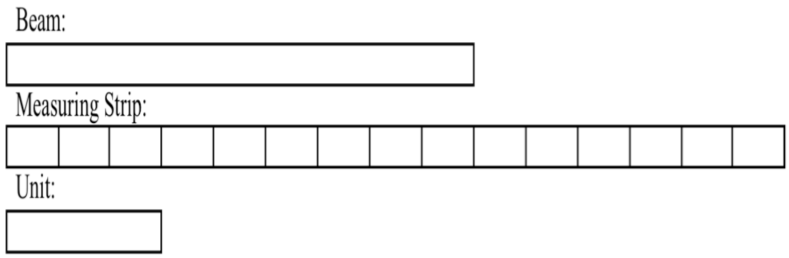

Figure 1: Context for the task

The activity she used to solve these tasks was the following. First, she measured the quantity with the long strip to determine the number of pieces in the quantity. She then measured the unit with the long measuring strip to determining the size of pieces. Because the earlier work had begun with mixed numbers, Kylie tended to give the answer as a mixed number either using division or more informal strategies. However, she was also able to give the answer as an improper fraction.

Kylie: It is two and one fifth.
Researcher: Or?
Kylie: Eh or eleven fifths.

In this example, we see she can anticipate what the quantity will be when it is grouped into units. She does not have to measure with multiple units. She can anticipate what the result of her activity of measuring with the unit would be given that she knows the number of partial units in the quantity and the number of partial units in the unit

After completing several tasks and demonstrating an anticipation of the results, she was asked a similar question but without the context.

Researcher: Okay. How about if I told you I had something that was ... thirteen thirds. Could you tell me another way to say it?
Kylie: (pause) Three ... no. Ten and three thirds? …
Researcher: Okay. Why do you say ten and three thirds?
Kylie: … cause there's ten … well, (pause) well, ten units and three thirds.
Researcher: (pause) you're just looking at these numbers separately?
Kylie: Yeah. Mhmm.
Researcher: Okay.
Kylie: Thirty-one thirds?

Although she had just demonstrated that she could anticipate what would happen if she measured an improper fraction like thirteen-thirds with a unit, she could not answer this question. In order to reconcile this disparity, we used the participatory-anticipatory distinction. The distinction suggests that two seemingly identical tasks could in fact demand different levels of abstraction, that is, different anticipations. We examined the distinction between the latest task and the previous ones in order to begin to articulate the differences in understanding. In the previous tasks, her attention was directed to the bar on the screen. Although she did not need to complete the activity in order to anticipate the result, she was cued to think about the activity. She saw the onscreen situation as being a question of measuring the unit and fractional parts of the unit. In the most recent question, she was not prompted to think about these

particular measurement activities. In fact there was no measurement expressly asked for. We will present a detailed analysis of the tasks Kylie could not do after we present some additional data

Converting Mixed Numbers to Improper Fractions

This same phenomenon happened when Kylie was engaged in finding equivalent improper fractions from mixed numbers. In these tasks, was given a unit and asked to make a bar of a given length, such as three and two-fifths. She would create this quantity by iterating the unit three times, partitioning another unit into five pieces, pulling out two of the pieces and joining them to the three unit bar. After the creation of the quantity, the researcher asked how long the bar would be if it were cut into fifths. Her activity in these tasks involved partitioning each of the units into five pieces and then counting all of the pieces. When the numbers became cumbersome to count, she used multiplication to help her determine the number of the pieces. In addition, after she completed the initial task, the language used by the researcher changed so that she was asked what improper fraction the quantity was equal to, instead of how long it would be if it were cut into pieces. The use of the term improper fraction did not seem to confuse her or change her activity.

After completing several of these tasks, she began to anticipate the answer without partitioning the units. In the excerpt below, the researcher had made a bar that was five and five sixths units long.

Researcher: Do you know what improper fraction it's equal to?
Kylie: Uh ... (pause) If it was all cut up into...sixths?
Researcher: If this were all sixths, how many would there be?
Kylie: (pause) Oh! Thirty-five ...
Researcher: Why thirty-five?
Kylie: Cause there's five sixths over here, and if it was all cut up, this part would be thirty and then that would be five.
Researcher: Okay. How do you know this part would be thirty?
Kylie: Cause each one of these is six.
Researcher: Uh huh.
Kylie: Six, twelve, eighteen, twenty-four….
Researcher: How do you know each one of those is six?
Kylie: Cause this is sixths.

In this excerpt, she can anticipate the result of cutting the bars into sixths without having to perform the activity.

After successfully demonstrating she could anticipate the results of partitioning a mixed number on several tasks, she was asked the following.

Researcher: I have a candy bar that's ten and a third units long. Can you tell me what the improper fraction would be?
Kylie: (pause) ten thirds? Ten... (pause) ten ...

Similar to what happened earlier, she suddenly is unable to answer the question.

Enlisting the Participatory-Anticipatory Distinction

An implication of the participatory-anticipatory distinction is that if a difference in performance can be attributed to this distinction, the tasks must have demanded a different level of abstraction from the learner. By *task*, we must consider not just the written or oral articulation of the task, but its position in a sequence of tasks and the tools available to solve the task. Let us first look closely at the tasks in this section that Kylie was able to do correctly. Kylie was given (or asked to draw) a bar representation of the mixed number. She was then asked to identify the improper fraction equivalent, which she understood as the number of fractional parts if the whole units were also broken up so the whole bar was partitioned into equal parts. Through her activity of partitioning the whole units and determining the total number of fractional parts, she came to be able to anticipate the number without actually partitioning the unit. Thus,

the tasks prompted her to look at the fractional part, consider the number of parts that the wholes would be broken into and then total the parts—often through multiplication followed by addition. To summarize, Kylie was carrying out the task of finding the number of partial units that would measure a bar that was initially measured in both whole units and partial units.

Did the last task (candy bar of length 10 1/3 units) require the same level of abstraction? Or did this task require an anticipatory level of knowing, whereas the prior tasks required only a participatory level? We argue for the usefulness of the latter. In this last task Kylie was asked to convert from a mixed number to an improper fraction. How was this different? Wasn't that what she was doing before? No. Before, the mixed number specified a bar to draw (sometimes drawn by the researcher), and then determine how many partial units were in that bar. Her focus was not on the equivalence of two representations. In the last task, she was asked to change a number written one way into a number written another way. She needed to know (to anticipate) to call on her prior activity, that is anticipate that if she thought about drawing a bar and partitioning it, that she would know the number of partial units and therefore the improper fraction. However, this was an anticipation that she had not yet developed. The following arrow diagram represents this claim.

Initially, Kylie developed an anticipation of the effect of her activity sequence: $A \rightarrow E$ (A is the activity, E is the effect). This anticipation was learned in a particular type of task for which the learner had a particular goal (e.g., determine the number of partial units in the bar on the screen). However the concept that was being developed (the researcher's instructional goal), represented by the last (anticipatory) task, required that she anticipate the need to call on that activity in response to a task that differed from the task through which the original anticipation ($A \rightarrow E$) was developed. This new anticipation that was needed can be represented as the relationship between a new goal G_1 and the activity A which is already linked to effect E by the original anticipation. Thus, the anticipatory stage requires the anticipation represented as $G_1 \rightarrow (A \rightarrow E)$.

Kylie did not have the anticipation between G_1 and the activity. She did not know (had not developed the anticipation) to call on her partitioning-and-totaling activity sequence in response to this equivalence question. When Kylie responded 10 1/3 = 10/3, the researcher created a bar that was 10 and 1/3 units long and prompted her to use the bars to see if she was correct.

Researcher: Okay. So you said, you said it's ten thirds, right?
Kylie: Mmhmm
Researcher: Figure it out, is that ten thirds?
Kylie: No.
Researcher: How much is it?
Kylie: It's ... (pause) one two three. Uh, three, six, nine, twelve, fifteen, eighteen … twenty- one, twenty-four ... (pause) twenty-seven? (pause) thirty. Thirty-one. It's thirty-one thirds.

The data excerpt above is consistent with the analysis. When prompted for the activity, she can make use of her anticipation regarding the number of partial units. We see she can successfully anticipate the results when prompted to think about the activity. It is clear that the anticipation required for the two tasks is different. When prompted to think about the activity, she needs to anticipate the results of the activity. When given a task that does not explicitly refer to the activity, she needs to anticipate the activity she needs to call upon. These two stages require distinct levels of understanding. It is clear she has the first anticipation, while the second type of abstraction remains outside of her current understanding. We leave it to the reader to make a similar argument relative to the first data segment (improper fraction to mixed number).

The diagram below (see Figure 2) is meant to represent Kylie's knowledge. The vertical arrows represent the anticipation that Kylie developed about the relationship between the diagram showing a mixed number and its equivalent improper fraction. The right arrow shows the reverse. The question Kylie is focused on for each vertical arrow has to do with the size of the bar. The dotted arrow reflects the lack of anticipation of the relationship between improper fractions and mixed numbers in response to the create-the-equivalent question.

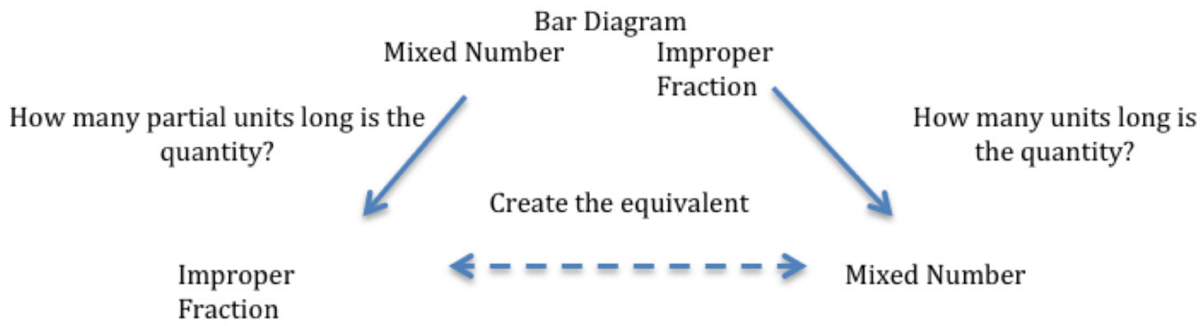

Figure 2: Diagram of Kylie's knowledge

The following data segment further strengthens the analysis. Kylie watched the researcher make a candy bar on the screen that was 2 and 5/7 units long. He then directed her the paper in front of her.

Researcher: Okay. How many... can you tell me two and five sevenths as ... (writing) as a, as an improper fraction?
Kylie: (pause) um. (pause) … twenty-five sevenths?
Researcher: Twenty five sevenths, look up there, does that look like twenty-five sevenths?
Kylie: No.
Researcher: What does it look like?
Kylie: … seven, fourteen … fifteen, sixteen, seventeen, eighteen, nineteen, nineteen sevenths.
Researcher: Nineteen sevenths.

Although the bar was on the screen in front of her, Kylie did not use it. One cannot be sure that she was aware that it was the length indicated on the paper. However, her failure to consider it when answering the question can be seen as her inability to connect the task and the activity she had used.

Conclusion

The data analysis provided demonstrates the explanatory power of differentiating between the anticipatory and participatory stages of conceptual learning. In Tzur and Simon (2004), the distinction was demonstrated with "the next day phenomenon," a common experience of educators. These data are even more compelling as the contrasting problems come one after the other in the same session (and with multiple examples). Without this construct, we would have struggled with understanding what seemed like inconsistent knowing on Kylie's part.

Researchers often expect that newly learned concepts might be inconsistently called on (Siegler, 1995). However, what percentage of those situations could be explained by this distinction? The distinction allows us to observe, analyze and conceptualize stages of learning a particular concept. It permits us to generate more thorough accounts of student learning. In some cases, we can use the distinction to anticipate aspects of a hypothetical learning trajectory (Simon, 1995) and in other cases, it allows us to notice in the data when we have failed to anticipate the challenge of moving from a participatory level to an anticipatory level. Explaining a data sequence as a move from participatory to anticipatory has a significant effect on task design in our teaching experiments. We can design tasks that aim directly at the new anticipation needed, the anticipation of the activity (linked to the effect) in response to the new goal. Continuing to provide experience at the participatory level would not benefit the student.

We close with a note about the use of the participatory-anticipatory distinction. The claim that an anticipation is at a participatory or anticipatory stage is relative to the particular concept in question and the related learner goal and activity. Thus, Kylie's ability to look at a bar measured in whole and partial units and anticipate the measurement in partial units only is neither participatory nor anticipatory. Rather it

is useful to think of it as participatory relative to understanding conversion of mixed numbers to improper fractions, the goal of making the conversion, and the activity of partitioning whole units into partial units.

Endnotes

[1] This was done on a computer using JavaBars (Biddlecomb & Olive, 2000). The focus in these activities was only on length (horizontal dimension).

[2] This paper is based upon work supported by the National Science Foundation under Grant No. DRL-1020154. Any opinions, findings, and conclusions or recommendations expressed in this paper are those of the authors and do not necessarily reflect the views of the National Science Foundation.

[3] Originally, this work was based on a specific elaboration referred to as "reflection on activity-effect relationships" (Simon et al., 2004). Whereas Tzur has continued to work with that elaboration, Simon has chosen to embark on a program of research, using the underlying concepts of reflection, activity, and abstraction, to conduct particularly rigorous teaching experiments (see Simon et al., 2010) to build a strong empirical base for elaborating a mechanism or mechanisms for conceptual learning.

References

Biddlecomb, B., & Olive, J. (2000). JavaBars [Computer software]. Retrieved September 1, 2011, from http://math.coe.uga.edu/olive/welcome.html

Davydov, V. V., Gorbov, S., Mikulina, T., & Savel'eva, M. (1995). *Manual for teachers*. Moscow: Moscow Press. [Translated, 2000]

Davydov, V. V., Gorbov, S., Mukulina, T., Savelyeva, M., & Tabachnikova, N. (1999). *Mathematics*. Moscow: Moscow Press. [Translated, grades 1–3, 2000]

Piaget, J. (2001). *Studies in reflecting abstraction* (R. L. Campbell, Ed. & Trans.). Philadelphia, PA: Psychology Press.

Siegler, R. S. (1995). How does change occur: A microgenetic study of number conservation. *Cognitive Psychology,* 28(3), 225–273.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*, 114–145.

Simon, M. A., Saldanha, L., McClintock, E., Karagoz Akar, G., Watanabe, T., & Ozgur Zembat, I. (2010). A developing approach to studying students' learning through their mathematical activity. *Cognition and Instruction,* 28, 70–112.

Simon, M. A., Tzur, R., Heinz, K., & Kinzel, M. (2004). Explicating a mechanism for conceptual learning: Elaborating the construct of reflective abstraction. *Journal for Research in Mathematics Education, 35*, 305–329.

Tzur, R. (2007). Fine grain assessment of students' mathematical understanding: Participatory and anticipatory stages in learning a new mathematical conception. *Educational Studies in Mathematics*, 66, 273–291.

Tzur, R., & Lambert, M. A. (2011). Intermediate participatory stages as zone of proximal development correlate in constructing counting-on: A plausible conceptual source for children's transitory "Regress" to counting-all. *Journal for Research in Mathematics Education, 42*(5), 418–450

Tzur, R., & Simon, M. A. (2004). Distinguishing two stages of mathematics conceptual learning. *International Journal of Science and Mathematics Education*, 2, 287–304.

von Glasersfeld, E. (1995). Radical constructivism: A way of knowing and learning. London: Falmer.